Prove that $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\cdots+\frac{1}{(3n-2)\times(3n+1)}=\frac{n}{3n+1}$ for all integers $n\geq1$ **SCORE: _____ / 10 PTS**

<u>by mathematical induction, showing all steps demonstrated in lecture.</u>

BASIS STEP:

If $n=1$, $\frac{1}{1\times4}=\frac{1}{4}$ $\qquad \frac{1}{3(1)+1}=\frac{1}{4}$ 1

INDUCTIVE STEP:

1 Assume $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\cdots+\frac{1}{(3k-2)\times(3k+1)}=\frac{k}{3k+1}$ for some arbitrary but particular integer k 1

2 $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\cdots+\frac{1}{(3k-2)\times(3k+1)}+\frac{1}{(3k+1)\times(3k+4)}$

1 $=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}$ FIRST & LAST 2 TERMS MUST BOTH BE SHOWN

$=\frac{k(3k+4)+1}{(3k+1)(3k+4)}$

1 $=\frac{3k^2+4k+1}{(3k+1)(3k+4)}$

1 $=\frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$

$=\frac{k+1}{3k+4}$

1 $=\frac{k+1}{3(k+1)+1}$

1 By mathematical induction, $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\cdots+\frac{1}{(3n-2)\times(3n+1)}=\frac{n}{3n+1}$ for all integers $n\geq1$

Evaluate $\sum_{k=1}^{300}(10k-3k^2)$. **Your final answer must be a number (not involving arithmetic operations).** SCORE: _____ / 4 PTS

$$= 10\sum_{k=1}^{300} k - 3\sum_{k=1}^{300} k^2 \quad 1$$

$$= 10 \cdot \tfrac{1}{2}(300)(301) - 3 \cdot \tfrac{1}{6}(300)(301)(601) \quad 1 \quad 1$$

$$= -26683650 \quad 1$$

If $f(x)=x^4$, expand and **completely simplify** the difference quotient $\dfrac{f(x+h)-f(x)}{h}$. $2\frac{1}{2}$ SCORE: _____ / 4 PTS

$$\frac{(x+h)^4-x^4}{h} = \frac{\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4-\cancel{x^4}}{h}$$

$$= 4x^3+6x^2h+4xh^2+h^3 \quad 1\tfrac{1}{2}$$

Expand and **completely simplify** the complex number $(3-2i)^5$. SCORE: _____ / 7 PTS

$$1(3)^5(-2i)^0+\underbrace{5(3)^4(-2i)^1}_{\frac{1}{2}}+\underbrace{10(3)^3(-2i)^2}_{\frac{1}{2}}+\underbrace{10(3)^2(-2i)^3}_{\frac{1}{2}}+\underbrace{5(3)^1(-2i)^4}\ \tfrac{1}{2}$$
$$+1(3)^0(-2i)^5$$

$$= \overset{\frac{1}{2}}{\underline{243}}+5(81)(-2i)+10(27)(-4)+10(9)(8i)+5(3)(16)$$
$$\underline{-32i}\ \tfrac{1}{2}$$

$$= 243\ \overset{\frac{1}{2}}{\underline{-810i}}\ \overset{\frac{1}{2}}{\underline{-1080}}\ \overset{1}{\underline{+720i}}\ \overset{1}{\underline{+240}}\ -32i$$

$$= \underline{-597-122i}\ \ 1$$

Find the 7^{th} term of $(8b-11g)^{25}$. **Your final coefficient may be in factored form as shown in lecture.** SCORE: _____ / 5 PTS

$${}_{25}C_6\,(8b)^{25-6}(-11g)^6$$

$$= \frac{25!}{6!\,19!}\,\overset{\frac{1}{2}}{\underline{(8b)^{19}}}\,\underline{(-11g)^6}\ \tfrac{1}{2}$$

PLUS $\frac{1}{2}$ IF YOUR FINAL ANSWER IS POSITIVE

$$= 1\ \boxed{\frac{25\cdot\cancel{24}\cdot23\cdot22\cdot\overset{7}{\cancel{21}}\cdot\overset{2}{\cancel{20}}\cdot\cancel{19!}}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1\cdot\cancel{19!}}}\,8^{19}\,11^6\,b^{19}g^6$$

$$= \tfrac{1}{2}\ \underline{25\cdot23\cdot22\cdot7\cdot2}\cdot8^{19}\cdot11^6b^{19}g^6 = 177100\cdot\overset{\frac{1}{2}}{\underline{8^{19}}}\cdot\overset{\frac{1}{2}}{\underline{11^6}}\ \overset{1}{\underline{b^{19}g^6}}$$